JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 19, No. 4, July–August 1996

Optimized Feedforward Design Scheme Unifying Regulator and Command Generator Tracker

Dohyeon Kim* and Youdan Kim†
Seoul National University, Seoul 151-742, Republic of Korea

Tracking control law is the control scheme to make the system output follow a nonzero reference command signal. By combining the tracking control law with an eigenstructure assignment and applying it to an advanced aircraft such as control configured vehicles, mode-decoupled command following can be achieved. The tracking control law usually consists of feedback control of output states and feedforward control of reference input. A new design methodology is proposed by utilizing the nonlinear optimization technique to design a useful controller. The proposed algorithm utilizes the eigenstructure assignment and the optimization technique to compute the feedback gain matrix (for aircraft mode decoupling) and feedforward gain matrix, respectively. It is shown that the existing algebraic formula for computing the feedforward gain matrix is the limiting case of the proposed methodology. The design methodology is illustrated by application to an AFTI F-16 aircraft.

I. Introduction

FOR a multi-input/multi-output system, a given set of eigenvalues can be assigned by an infinity of gain matrices. The eigenstructure assignment technique utilizes this extra freedom to assign the closed-loop eigenvectors. The original eigenstructure assignment technique based on the algebraic solution is very useful for aircraft mode decoupling.[1–3] The main drawback of this technique is the lack of stability robustness with respect to parameter variations. To solve this problem, research[4–7] has been performed using optimization[4–6] or the method of inequality.[7] In this paper, we are not very concerned with these feedback methodologies; thus, we just adopt the baseline eigenstructure assignment technique. Sobel and Shapiro[2] developed the design methodology for pitch pointing flight control systems by utilizing an eigenstructure assignment in conjunction with the command generator tracker.[8] This methodology computes the feedback gain matrix by eigenstructure assignment, then the feedforward gain matrix can be obtained by solving an algebraic unsymmetric Lyapunov equation. Therefore, this method does not allow the designer to use his own perspective for computing the feedforward gain matrix during the design process.

In this paper, we propose a new design methodology for computing the feedforward gain matrix. The feedforward gains are computed to minimize a performance index, which considers both the error deviation from the prescribed model dynamics and steady-state error. By selecting the design parameters, the feedforward gain matrix will yield various tracking performances. We prove that the perfect model following control scheme is one of the limiting case of the proposed design methodology. The proposed design schemes are illustrated by application to an AFTI F-16 aircraft for pitch pointing control.

II. Model Following Control Scheme

It is well known that the pitch pointing flight control laws can be designed by using the model following control scheme utilizing an eigenstructure assignment and command generator tracking.[1,2] In this section we summarize the model following design methodology developed by Andry et al.[1] and Sobel and Shapiro.[2]

Presented as Paper 95-3265 at the AIAA Guidance, Navigation, and Control Conference, Baltimore, MD, Aug. 7–10, 1995; received Sept. 18, 1995; revision received April 8, 1996; accepted for publication April 9, 1996. Copyright © 1996 by the American Institute of Aeronautics and Astronautics, Inc. All rights reserved.

*Graduate Student, Department of Aerospace Engineering. Student Member AIAA.

†Assistant Professor, Department of Aerospace Engineering. Member AIAA.

Problem Statement

Consider the linear time-invariant system described by

$$\dot{x} = Ax + Bu \tag{1}$$

$$y = Cx \tag{2}$$

$$z = Hx + Gu \tag{3}$$

where $x(t) \in R^n$ the state, $u(t) \in R^m$ the control input, $y(t) \in R^r$ the measured output available for the feedback purpose, and $z(t) \in R^s$ the performance (or controlled) output that must track the given reference input. The performance output $z(t)$ is not generally equal to $y(t)$. We shall assume that rank$[B] = m$ and rank$[C] = r$.

Let us consider an output feedback control of the form

$$u = Fy + s(t) \tag{4}$$

where F denotes the feedback gain matrix and $s(t)$ is an input that might be used for making the performance output track the reference input. By substituting the control into the state equation, the closed-loop system is obtained:

$$\dot{x} = (A + BFC)x + Bs \equiv A_c x + Bs \tag{5}$$

Feedback Design Methodology

The feedback gain matrix F can be selected for assigning closed-loop eigenvalues λ_i and associated eigenvectors to the desired values. In this study, we adopted the eigenstructure assignment technique[1–3] for designing feedback control gain and briefly summarize the design procedure here. We only consider square nonsingular C matrix (mathematically the same as the state feedback case), because the assignability computation may digress the main point, i.e., the feedforward design scheme. First, for each desirable eigenvalue λ_i, find the null space basis set of the augmented matrix

$$\ker[(A - \lambda_i I) \quad B] = \text{span}\begin{bmatrix} v_{i1} & v_{i2} & \cdots & v_{im} \\ w_{i1} & w_{i2} & \cdots & w_{im} \end{bmatrix} \tag{6}$$

where ker denotes the kernel space of the given matrix and v_{ij} and w_{ij} are n- and m-dimensional vectors, respectively. The following vectors can be generated by linear combinations of vectors v_{ij} and w_{ij}:

$$v_i = \sum_{j=1}^{m} \alpha_{ij} v_{ij} \tag{7}$$

$$w_i = \sum_{j=1}^{m} \alpha_{ij} w_{ij} \tag{8}$$

Because vectors v_i and w_i are obtained by solving Eq. (6), it is natural that they satisfy

$$(A - \lambda_i I)v_i + Bw_i = 0 \tag{9}$$

Let us define the matrices V and W

$$\begin{aligned} V &= [v_1 \quad v_2 \quad \cdots \quad v_n] \\ W &= [w_1 \quad w_2 \quad \cdots \quad w_n] \end{aligned} \tag{10}$$

Comparing the eigenvalue problem of the closed-loop system Eq. (5) with Eq. (9) leads to the conclusion that the feedback gain matrix can be computed by the following equation:

$$F = W(CV)^{-1} \tag{11}$$

and vector v_i defined in Eq. (7) is the associated closed-loop eigenvector to the ith eigenvalue λ_i. Therefore, the coefficients α_{ij} in Eqs. (7) and (8) should be selected in consideration of the form of the desired eigenvector. Note that because the closed-loop eigenvectors must be linearly independent, it is necessary to select v_i as linearly independent vectors.

Feedforward Design Methodology

To make the system follow the prescribed model dynamics, it is necessary to include the feedforward of model states in the control input. In this subsection, we shall briefly derive the widely used feedforward design methodology known as the command generator tracker.[2,8] Suppose that the model, which describes the desired behavior of an aircraft, is described by

$$\begin{aligned} \dot{x}_m &= A_m x_m + B_m u_m \\ z_m &= H_m x_m + G_m u_m \end{aligned} \tag{12}$$

Let the state and control vectors be represented by model variables and error vectors (such as high-order derivatives) as follows:

$$\begin{aligned} x &= S_{11} x_m + S_{12} u_m + \delta x \\ u &= S_{21} x_m + S_{22} u_m + \delta u \end{aligned} \tag{13}$$

where S_{ij} are constant matrices that will be determined so that the performance output $z(t)$ closely approximates the model output $z_m(t)$. The model input u_m is the pilot's command for the aircraft control problem and is restricted to be a step input. With this assumption, using Eqs. (1), (3), (12), and (13) and applying the model matching condition, $z = z_m + \delta z$, yield the ideal control input as follows[8]:

$$u = FCx + (S_{21} - FCS_{11})x_m + (S_{22} - FCS_{12})u_m \tag{14}$$

with

$$\begin{bmatrix} S_{11} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A_m & B_m \\ H_m & G_m \end{bmatrix} = \begin{bmatrix} A & B \\ H & G \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \tag{15}$$

In this study, our design objective is to make the aircraft's performance output track the pilot's command. This model can be obtained by considering the pilot's command as the model's output, and this identity model is described by[2]

$$A_m = 0, \qquad B_m = 0, \qquad H_m = 0, \qquad G_m = I \tag{16}$$

Because the performance output z may be constructed by using only system states x, the matrix G in Eq. (3) can be taken as a zero matrix without loss of generality. Substitution of Eqs. (16) and $G = 0$ into Eqs. (15) leads to $S_{11} = 0$ and $S_{21} = 0$, if

$$\begin{bmatrix} A & B \\ H & 0 \end{bmatrix}$$

is invertible, and therefore, the control input becomes

$$u = FCx + F_f u_m \tag{17}$$

where F is the stabilizing gain matrix obtained by an eigenstructure assignment method discussed in the preceding section and F_f denotes the feedforward gain matrix obtained by

$$F_f = S_{22} - FCS_{12} \tag{18}$$

Note that the feedforward gain matrix F_f depends only on 1) the feedback gain matrix F, which is obtained by an eigenstructure assignment, and 2) constant matrices S_{12} and S_{22}, which are determined by Eq. (15). That means, after designing the feedback control gains for the given system, we do not have any design freedom for computing the feedforward gain matrix. Therefore, using the given control law, it is not easy to design controller for systems with bounded control inputs.

III. Optimal Feedforward/Feedback Design Methodology

Optimal Feedforward Design Methodology

In this section, a new design methodology for a feedforward gain matrix is proposed. By minimizing the norm of error for the model following, we derive a new formula that will be used for computing the feedforward gain matrix. There exists a tradeoff between the perfectness of the model following and the magnitude of the control input to be used, and the designer should select the weighting matrices to satisfy the design specifications. Let us denote the steady-state values by overbars and deviations from the steady-state values by carets. Then the state, output, and control deviations are given by

$$\begin{aligned} \hat{x} &= x - \bar{x} \\ \hat{y} &= y - \bar{y} = C\hat{x} \\ \hat{u} &= u - \bar{u} = Fy + F_f u_m - F\bar{y} - F_f \bar{u}_m \\ &= F\hat{y} + F_f(u_m - \bar{u}_m) = FC\hat{x} + F_f \hat{u}_m \end{aligned} \tag{19}$$

where the model state, input deviations are defined as

$$\begin{aligned} \hat{x}_m &= x_m - \bar{x} \\ \hat{u}_m &= u_m - \bar{u} \end{aligned} \tag{20}$$

The tracking error $e(t)$ is given by

$$e = z_m - z \tag{21}$$

with the error deviation given by

$$\begin{aligned} \hat{e} = e - \bar{e} &= (H_m x_m + G_m u_m - Hx - Gu) \\ &\quad -(H_m \bar{x}_m + G_m \bar{u}_m - H\bar{x} - G\bar{u}) \\ &= H_m \hat{x}_m + G_m \hat{u}_m - H\hat{x} - G\hat{u} \end{aligned} \tag{22}$$

Using Eqs. (1), (2), and (17), at steady state

$$\bar{x} = A_c^{-1} B F_f \bar{u}_m \tag{23}$$

For the asymptotically stable system, the system states approach the prescribed model states; therefore the steady-state system states can be considered as the steady-state model states (i.e., $\bar{x} = \bar{x}_m$), and the steady-state error becomes

$$\begin{aligned} \bar{e} &= \bar{z}_m - \bar{z} \\ &= H_m \bar{x}_m + G_m \bar{u}_m - H\bar{x} - G\bar{u} \\ &= (H_m - H)\bar{x} + [G_m \bar{u}_m - G(FC\bar{x} + F_f \bar{u}_m)] \\ &= (H_m - H - GFC)\left(-A_c^{-1} BF_f\right)\bar{u}_m + (G_m - GF_f)\bar{u}_m \\ &= \left[-(H_m - H - GFC)A_c^{-1} BF_f + (G_m - GF_f)\right]\bar{u}_m \end{aligned} \tag{24}$$

To make the tracking error $e(t)$ small, let us take the performance index that minimizes the error deviation $\hat{e}(t)$ and the steady-state error $\bar{e}$ simultaneously as follows:

$$J = \frac{1}{2}\int_0^\infty (\hat{e}^T Q\hat{e} + \hat{u}^T R\hat{u})\,\mathrm{d}t + \frac{1}{2}\bar{e}^T V\bar{e} \tag{25}$$

where $Q \geq 0$, $R > 0$, and $V \geq 0$ are the weighting matrices. Note that making the error deviation $\hat{e}(t)$ small improves the transient response, whereas making the steady-state error $\bar{e}$ small improves the steady-state response.

To make the aircraft's performance output track the pilot's command, our model can be simplified as in Eq. (16). Also, for the step model input u_m, $\hat{u}_m$ in Eq. (20) is zero and the performance index in Eq. (25) is also simplified. By applying optimal control theory[9] to the resulting equation, we obtained the following algebraic Riccati equation:

$$f \equiv A_c^T P + PA_c + H^T QH + C^T F^T RFC = 0 \tag{26}$$

In this study, the Riccati equation may be interpreted in a different way, because our design objective is to obtain the optimal feedforward gains that yield various tracking performance for the given feedback gains. Note that the feedback gain matrix F is previously computed by an eigenstructure assignment and the unknown in Eq. (26) is matrix P. Therefore, in our formulation the optimal feedforward gain matrix F_f will be determined through solution P.

By using a technique like the one used in the optimal output feedback control system design,[9,10] the optimal cost is found to satisfy

$$J = \tfrac{1}{2}\,\hat{x}^T(0)P\hat{x}(0) + \tfrac{1}{2}\,\bar{e}^T V\bar{e} \tag{27}$$

where $P > 0$ is the solution of Eq. (26). For the tracking problem, we assume that the system starts at rest [i.e., $x(0) = 0$] and

$$\hat{x}(0) = x(0) - \bar{x}(0) = -\bar{x} \tag{28}$$

Therefore, the optimal cost Eq. (27) becomes

$$J = \tfrac{1}{2}\mathrm{tr}\{PX\} + \tfrac{1}{2}\,\bar{e}^T V\bar{e} \tag{29}$$

with P given by Eq. (26), $\bar{e}$ given by Eq. (24), and

$$\begin{aligned} X &= \hat{x}(0)\hat{x}(0)^T = \bar{x}\bar{x}^T \\ &= A_c^{-1}BF_f\bar{u}_m\bar{u}_m^T F_f^T B^T A_c^{-T} \end{aligned} \tag{30}$$

To solve this modified problem, we use the Lagrange multiplier approach.[9] Adjoin the constraint equations (26) and (30) to the performance index, Eq. (29), by defining the Hamiltonian

$$\begin{aligned} \mathcal{H} &= \tfrac{1}{2}\,\mathrm{tr}\left\{PA_c^{-1}BF_f\bar{u}_m\bar{u}_m^T F_f^T B^T A_c^{-T}\right\} + \mathrm{tr}\{fS\} \\ &\quad + \tfrac{1}{2}\left\{\bar{u}_m^T\left(I + HA_c^{-1}BF_f\right)^T V\left(I + HA_c^{-1}BF_f\right)\bar{u}_m\right\} \end{aligned} \tag{31}$$

where f is defined by Eq. (26) and S is a symmetric matrix of Lagrange multipliers. Now our problem is to find the feedforward gain F_f that minimizes Eq. (31) without constraints, and the optimal condition can be obtained by setting the partial derivatives of $\mathcal{H}$ with respect to F_f equal to zero,

$$\frac{\partial\mathcal{H}}{\partial F_f} = \left\{B^T A_c^{-T}(P + H^T VH)A_c^{-1}BF_f + B^T A_c^{-T}H^T V\right\}\bar{u}_m\bar{u}_m^T = 0 \tag{32}$$

Even though $\bar{u}_m\bar{u}_m^T$ is a singular matrix, the sufficient condition for satisfying the Eq. (32) can be obtained as follows:

$$F_f = -\left\{B^T A_c^{-T}(P + H^T VH)A_c^{-1}B\right\}^{-1} B^T A_c^{-T}H^T V \tag{33}$$

Equation (33) will be used for computing the feedforward gain matrix.

Some Physical Interpretations: Limiting Cases

By selecting the judicious weighting matrices Q, R, and V, we can obtain the feedforward gain matrix that yields good tracking performance. It is useful to study the limiting cases of the preceding three design parameters, and this will provide us the guideline for choosing the appropriate weighting matrices.

First, consider the limiting case of weighting matrix R. For this case, the performance index, Eq. (25), can be rewritten as follows:

$$\frac{J}{\|R\|} = \frac{1}{2}\int_0^\infty \hat{x}^T\left(\frac{H^T QH}{\|R\|} + C^T F^T\frac{R}{\|R\|}FC\right)\hat{x}\,\mathrm{d}t + \frac{1}{2}\,\bar{e}^T\frac{V}{\|R\|}\bar{e} \tag{34}$$

For the proper Q and V matrices, the relative magnitude of matrix R is much larger than the matrices Q and V. From Eq. (26), the solution P becomes large for this limiting case, and therefore, the feedforward gain matrix that is obtained by Eq. (33) approaches zero:

$$F_f = 0 \tag{35}$$

Now, consider the limiting case of weighting matrix Q. It is easy to understand that the solution procedure of this case is similar to the former one, and again the zero feedforward gain matrix is obtained. For these limiting cases lead to a situation where they seek to make only the deviation state variables zero, without taking care of the steady-state error $\bar{e}$. By making the feedforward gain matrix F_f zero, the steady-state state variable becomes zero by Eq. (23). It is clear that the limiting case solution for this case is equivalent to the solution for the minimal energy problem, and the selection of Q and R should rely on engineering judgment for the system performance.

On the contrary, for the limiting case of weighting matrix V, the steady-state response is improved by minimizing the steady-state error. Using Eqs. (16) and (24), we obtain the necessary and sufficient condition for making the steady-state error zero:

$$I + HA_c^{-1}BF_f = 0 \tag{36}$$

Let us consider the perfect model following control scheme discussed in Sec. II. Using Eqs. (15) and (16), we obtain

$$AS_{12} + BS_{22} = 0 \tag{37}$$

$$HS_{12} = I \tag{38}$$

Substitution of $A = A_c - BFC$ and Eq. (37) into Eq. (38) yields

$$I + HA_c^{-1}B(S_{22} - FCS_{12}) = 0 \tag{39}$$

Comparison of Eqs. (36) and (39) leads to the conclusion that for the limiting case of the weighting matrix V the feedforward gain matrix will approach the solution $S_{22} - FCS_{12}$. In other words, the limiting solution for the extreme V will give us the same feedforward gain matrix as in the perfect model following control scheme.

Therefore, by selecting the proper weighting matrices, the obtained feedforward gain matrix will yield various tracking performances between 1) the regulator without tracking ability and 2) the perfect model following controller minimizing steady-state error. There is a design tradeoff involved in selecting the weighting matrices.

IV. Numerical Results

This example demonstrates the use of the optimal feedforward controller proposed in this paper. The design of the pitch pointing flight control system for the short-period motion of an AFTI F-16 aircraft is illustrated. The short-period approximation equations[2] augmented by control actuator dynamics at an air speed $M = 0.6$ with an altitude $h = 3000$ ft are described by

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

where the state vector comprises the flight path angle γ, the pitch rate q, the angle of attack α, the elevator deflection δ_e, and the flaperon deflection δ_f. The inputs to the system are the elevator deflection command δ_{ec} and the flaperon deflection command δ_{ef},

$$A = \begin{bmatrix} 0 & 0.00665 & 1.3411 & 0.16897 & 0.25183 \\ 0 & -0.86939 & 43.223 & -17.251 & -1.5766 \\ 0 & 0.99335 & -1.3411 & -0.16897 & -0.25183 \\ 0 & 0 & 0 & -20 & 0 \\ 0 & 0 & 0 & 0 & -20 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 20 & 0 \\ 0 & 20 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & -0.268 & 47.76 & -4.56 & 4.45 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The eigenvalues of the open-loop system are given by the following.

Unstable short period mode:

$$\left.\begin{aligned} \lambda_1 &= -7.662 \\ \lambda_2 &= 5.452 \end{aligned}\right\}$$

Pitch attitude mode:

$$\lambda_3 = 0$$

Elevator actuator mode:

$$\lambda_4 = -20$$

Flaperon actuator mode:

$$\lambda_5 = -20$$

Note that this advanced aircraft has an unstable open-loop pole. For stability, the desired closed-loop pitch attitude mode should decay exponentially with a time constant of 1, so that the pole should be at $s = -1$. The actuator poles should be near -20. The desired eigenvalues are thus selected as follows.

Short period mode:

$$\lambda_{1,2}^d = -5.6 \pm j\,4.2$$

Pitch attitude mode:

$$\lambda_3 = -1.0$$

Elevator actuator mode:

$$\lambda_4 = -19.0$$

Flaperon actuator mode:

$$\lambda_5 = -19.5$$

In pitch pointing, the control objective is to allow the pitch-attitude control while maintaining a constant flight-path angle. To accomplish pitch pointing, the desired eigenvectors are chosen as follows[3]:

$$\begin{matrix} \gamma \\ q \\ \alpha \\ \delta_e \\ \delta_f \end{matrix} \quad \underbrace{\begin{bmatrix} 0 \\ 1 \\ x \\ x \\ x \end{bmatrix} + i \begin{bmatrix} 0 \\ x \\ 1 \\ x \\ x \end{bmatrix}}_{\substack{\alpha/q \\ \text{short period}}} \quad \underbrace{\begin{bmatrix} 1 \\ 0 \\ x \\ x \\ x \end{bmatrix}}_{\gamma} \quad \underbrace{\begin{bmatrix} x \\ x \\ x \\ 1 \\ x \end{bmatrix}}_{\delta_e} \quad \underbrace{\begin{bmatrix} x \\ x \\ x \\ x \\ 1 \end{bmatrix}}_{\delta_f}$$

For this selection, the feedback gain matrix is obtained by an eigenstructure assignment discussed in Sec. II as follows:

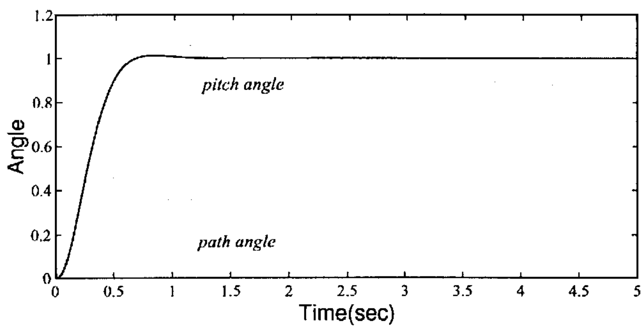
$$F = \begin{bmatrix} 0.9309 & 0.1489 & 3.2504 & 0.1530 & -0.7471 \\ -0.9542 & -0.2098 & -6.1008 & -0.5370 & 1.0352 \end{bmatrix}$$

Perfect Model Following Control Scheme

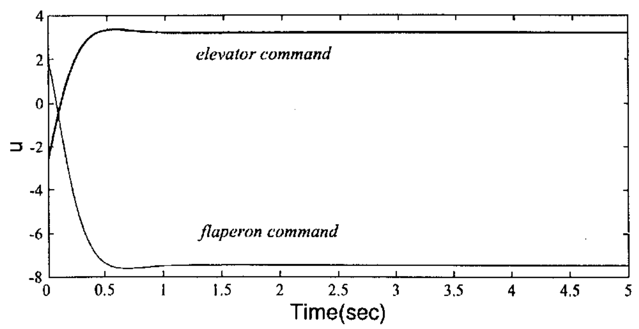
The computed feedforward gain matrix using the perfect model following scheme, Eq. (18), is

$$F_f = \begin{bmatrix} -2.8773 & -0.3731 \\ 1.9784 & 4.1244 \end{bmatrix}, \qquad \|F_f\|_{\mathrm{f}} = 5.4162$$

where $\|F_f\|_{\mathrm{f}}$ denotes the Frobenius norm of the feedforward gain matrix F_f. The pitch pointing responses are shown in Fig. 1.

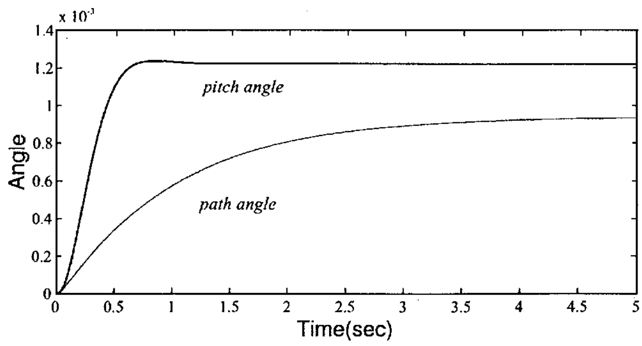


Pitch pointing response

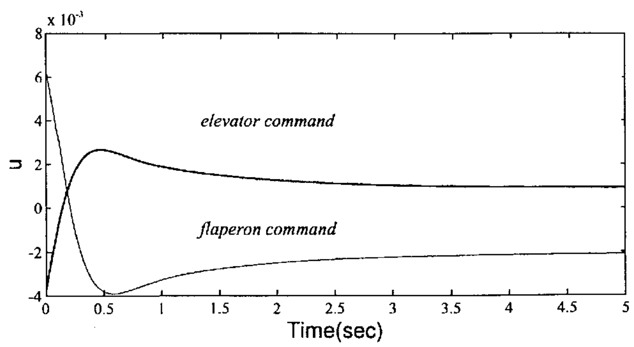


Control input

Fig. 1 Time response using algebraic feedforward.



Pitch pointing response



Control input

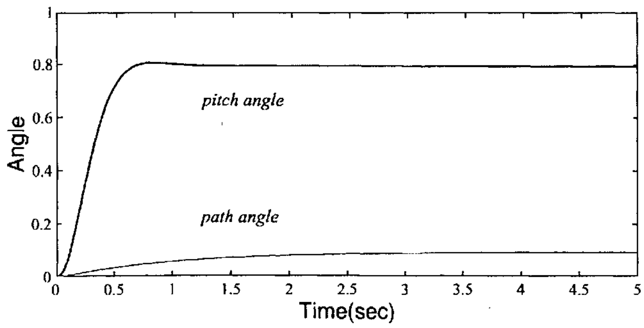
Fig. 2 Time response using optimized feedforward (R = 100 I, V = I, whereas $\bar{Q}$ fixed identity).

Table 1 **Feedforward gains for various weighting matrices ($Q = I$)**

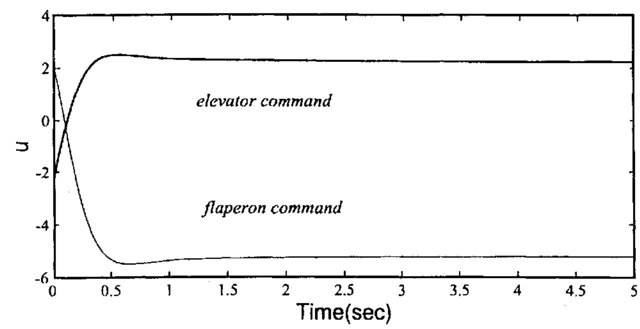
Cases	Feedforward gain matrix	Frobenius norm
Algebraic model following	$\begin{bmatrix} -2.8773 & -0.3731 \\ 1.9764 & 4.1224 \end{bmatrix}$	5.4162
Optimization ($R = 100I$, $V = I$)	$\begin{bmatrix} -0.0039 & -0.0034 \\ 0.0063 & 0.0093 \end{bmatrix}$	0.0124
Optimization ($R = I$, $V = 50I$)	$\begin{bmatrix} -2.3182 & -0.5878 \\ 1.9572 & 3.6837 \end{bmatrix}$	4.8084
Optimization ($R = 0.01I$, $V = \mathrm{diag}\{10^5, 10^3\}$)	$\begin{bmatrix} -2.8795 & -0.3697 \\ 2.0051 & 4.0847 \end{bmatrix}$	5.3796

Table 2 **Feedforward gains for various weighting matrices**

	Weighting matrices	Feedforward gain matrix
Case I	$Q = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}$	$F_f = \begin{bmatrix} -2.8784 & -0.2393 \\ 1.9954 & 2.6454 \end{bmatrix}$
	$R = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$	$\lVert F_f \rVert_F = 4.3956$
	$V = \begin{bmatrix} 100000 & 0 \\ 0 & 100 \end{bmatrix}$	
Case II	$Q = \begin{bmatrix} 1 & 0.999 \\ 0.999 & 1 \end{bmatrix}$	$F_f = \begin{bmatrix} -2.8626 & -0.3756 \\ 2.0180 & 4.0523 \end{bmatrix}$
	$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\lVert F_f \rVert_F = 5.3692$
	$V = \begin{bmatrix} 100000 & 0 \\ 0 & 100 \end{bmatrix}$	

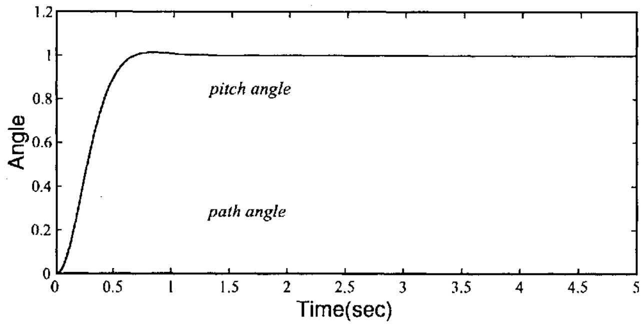


Pitch pointing response

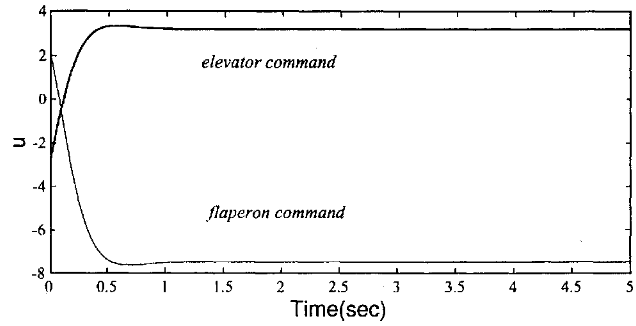


Control input

Fig. 3 Time response using optimized feedforward ($R = I$, $V = 50\,I$, whereas Q fixed identity).



Pitch pointing response



Control input

Fig. 4 Time response using optimized feedforward ($R = 0.01\,I$, $V = \mathrm{diag}\{10^5, 10^3\}$, $Q = I$).

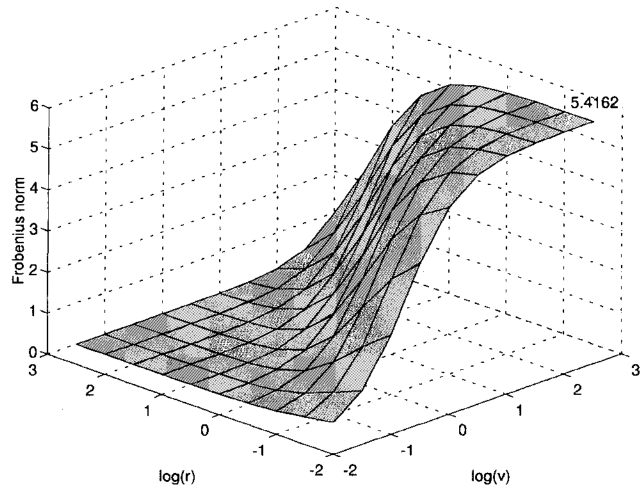


Fig. 5 Frobenius norm of feedforward gain matrix vs *r* and *v*. Magnitude of feedforward gain vs weightings.

Optimal Feedforward Design

A design procedure for optimal feedforward gain matrix would involve selecting the design parameters Q, R, and V. There are some guidelines on the selection of these weighting matrices, which we shall now discuss.

1) We can generally select Q, R, and V as

$$Q = qI, \qquad R = rI, \qquad V = vI$$

with I the identity matrix and q, r, and v scalar design parameters such as 10^n.

2) If the results are not satisfactory, then tune the scalar design parameters. One of the three design parameters may be fixed, since ratios between the design parameters are important.

3) Off-diagonal elements in weighting matrices can be added and tuned for the selected diagonal design parameters for further investigation.

Computed feedforward gain matrices for various weighting matrices with a fixed Q matrix are shown in Table 1, and the pitch pointing responses are shown in Figs. 2–4. The increased weighting V relative to weighting R has the effect of improving the model

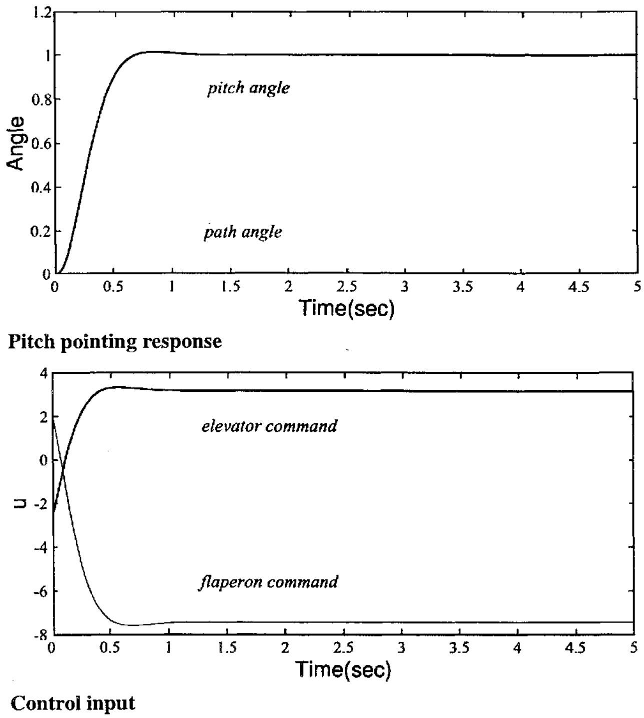


Fig. 6 Time response using optimized feedforward (R = 0.01I, V = diag$\{10^5, 10^2\}$, $\bar{Q}$ = 100I).

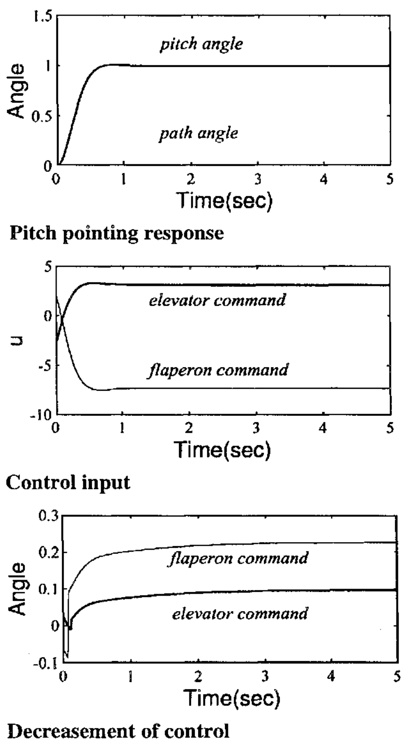


Fig. 7 Time response using optimized feedforward (R = I, V = diag$\{10^5, 10^2\}$, $\bar{Q}$ = diag$\{10^5, 10^2\}$).

following performance. Figures 2 and 3 show that relatively small V yields unacceptable tracking errors, while almost no controls are used in these cases. As shown in Figs. 1 and 4, for sufficiently large V the tracking performance is almost the same as that of the perfect model following control scheme. These results are expected by the limiting case analysis described in the preceding section. Figure 5 shows the Frobenius norm of the feedforward gain matrix vs weighting parameters r and v. Notice that the large r case produces zero feedforward gain, and large v case yields the same feedforward gain as the perfect model following control scheme. That is consistent with our expectation based on the analysis for the limiting cases discussed in the preceding section.

For the varying Q matrix case, the obtained feedforward gain matrices are shown in Table 2, and the pitch pointing responses are shown in Figs. 6 and 7. Figure 7 shows that for this particular example, by tuning the Q matrix, we obtain the controller having almost the same command tracking performance albeit consuming slightly smaller feedback control efforts. As discussed in the preceding section, these numerical examples show that by selecting the proper weighting matrices, we can obtain a control system with various tracking performances between 1) the regulator without tracking ability and 2) the perfect model following controller minimizing steady-state error.

V. Concluding Remarks

This paper has presented a new design methodology for computing the feedforward gain matrix by utilizing the optimal control theory. By selecting the proper weighting matrix, the designed controller exhibits various tracking performances between the regulator without tracking ability and the perfect model following controller minimizing steady-state error. An interesting feature is that the perfect model following control scheme is the limiting case of the proposed design methodology.

References

[1]Andry, A. N., Shapiro, E. Y., and Chung, J. C., "Eigenstructure Assignment for Linear Systems," *IEEE Transactions on Aerospace and Electronics*, Vol. AES-19, No. 5, 1983, pp. 715–729.

[2]Sobel, K. M., and Shapiro, E. Y., "A Design Methodology for Pitching Pointing Flight Control Systems," *Journal of Guidance, Control, and Dynamics*, Vol. 8, No. 2, 1985, pp. 181–187.

[3]Stevens, B. L., and Lewis, F. L., *Aircraft Control and Simulation*, Wiley, New York, 1992.

[4]Yu, W., and Sobel, K. M., "Robust Eigenstructure Assignment with Structured State Space Uncertainty," *Journal of Guidance, Control, and Dynamics*, Vol. 14, No. 3, 1991, pp. 621–628.

[5]Juang, J. C., Youssef, H. M., and Lee, H. P., "Robust of Eigenstructure Assignment Approach in Flight Control System Design," *Proceedings of the American Control Conference* (San Diego, CA), 1990, pp. 749–754.

[6]Burrows, S. P., and Patton, R. J., "Design of Low Sensitivity Modalized Observers Using Eigenstructure Assignment," *Journal of Guidance, Control, and Dynamics*, Vol. 15, No. 3, 1992, pp. 779–782.

[7]Patton, R. J., Liu, G. P., and Chen, J., "Multiobjective Controller Design Using Eigenstructure Assignment and the Method of Inequalities," *Journal of Guidance, Control, and Dynamics*, Vol. 17, No. 4, 1994, pp. 862–864.

[8]O'Brien, M. J., and Bloussard, J. R., "Feedforward Control to Track the Output of a Forced Model," *Proceedings of the 17th IEEE Conference on Decision and Control* (San Diego, CA), 1978, pp. 1149–1155.

[9]Lewis, F. L., *Applied Optimal Control and Estimation*, Prentice–Hall, Englewood Cliffs, NJ, 1992.

[10]Athans, M., and Levine, W. S., "On the Determination of the Optimal Constant Output Feedback Gains for Linear Multivariable Systems," *IEEE Transactions on Automatic Control*, Vol. AC-15, No. 1, 1970, pp. 44–48.